

# Statistics decomposition for NL Scoring (SD-NL)

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2020-11-17

# *What is NL?*

- Normalized likelihood
  - $p(x | H_0)$  : denotes as  $p_c(x)$  , which is a speaker-dependent item.
  - $p(x | H_1)$  : denotes as  $p(x)$  , which is a speaker-independent item.

$$NL(x | c) = \frac{p(x | H_0)}{p(x | H_1)} = \frac{p_c(x)}{p(x)}$$

# NL model

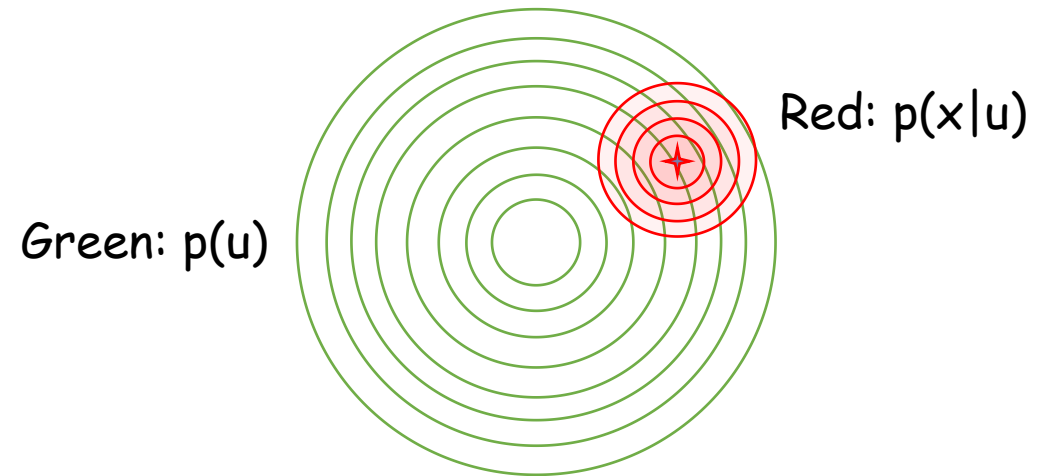
- A linear Gaussian model

$$p(u) = N(u; 0, \varepsilon I)$$

$$p(x | u) = N(x; u, \sigma I)$$

$$\begin{aligned} p(x) &= \int p(x | u) p(u) du \\ &= N(x; 0, (\varepsilon + \sigma) I) \end{aligned}$$

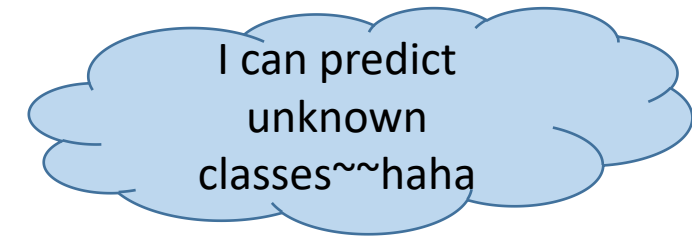
$$\begin{aligned} NL(x | u_c) &= \frac{p(x | u_c)}{p(x)} \\ &= \frac{p_c(x)}{p(x)} \\ &= \frac{p(x | x_1^c, x_2^c, \dots, x_n^c)}{p(x)} \end{aligned}$$



$$p(x) = \int p(x | u) p(u) du$$

# Three components of NL

$$NL(x|u_c) = \frac{p(x|u_c)}{p(x)} = \frac{p_c(x)}{p(x)} = \frac{p(x|x_1^c, \dots, x_n^c)}{p(x)} = \frac{\int p(x|u)p(u|x_1^c, \dots, x_n^c)du}{\int p(x|u)p(u)du}$$



- Decouple NL to **three** components
  - Enrollment :  $p(u|x_1^c, \dots, x_n^c)$  produces the posterior of class mean.
  - Prediction :  $p(x|u)$  computes the likelihood of x belonging to class c.
  - Normalization :  $p(x)$  computes the likelihood of x from all classes.

# Why we need decouple ?

- Background :
  - In practice, the data could be quite complex.
  - NL/PLDA is modeled by between-var and within-var, and it uses the same statistics for different components, which is obviously unreasonable.
- Ideal:
  - The different components use their own optimal model.
- So we need decouple, and :
  - A high-level and global perspective that overlooks the distributional of the entire data.
  - A low-level and local perspective that scrutinizes the distribution of a single class.

# How to implement decouple ?

- Enrollment  $p(u | x_1^c, \dots, x_n^c)$  and Normalization  $p(x)$  are relevant to a global generative model, e.g., PLDA.
  - $p_g(u) = N(u; 0, \epsilon I)$
  - $p_g(x | u) = N(x; u, \sigma I)$
- Predication  $p(x | u_c)$  regards as a local model
  - $p_l(x | u) = N(x; u, \Sigma')$

$$NL(x | u_c) = \frac{p(x | u_c)}{p(x)} = \frac{p_c(x)}{p(x)} = \frac{\int p_l(x | u) p_g(u | x_1^c, \dots, x_n^c) du}{\int p_g(x | u) p_g(u) du}$$

# Training process - Global

- Global training

- ML-PLDA

$$p(x_1, \dots, x_n) \propto |\sigma I|^{-\frac{n}{2}} |\varepsilon I|^{-\frac{1}{2}} \left( \frac{n}{\sigma} + \frac{1}{\varepsilon} \right) I \Big|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2\sigma} \left\{ \sum_i \|x_i\|^2 - \frac{n^2 \varepsilon}{n\varepsilon + \sigma} \|\bar{x}\|^2 \right\} \right\}$$

where  $|\cdot|$  defined is the absolute value of the determinant of a matrix. Given a training set consisting of K classes, the parameters  $\varepsilon$  and  $\sigma$  can be optimized by maximizing the likelihood function:

$$L(\varepsilon, \sigma) = \sum_{c=1}^C p(x_1^c, \dots, x_{n_c}^c)$$

where  $x_i^c$  is the i-th sample of the c-th class.

- ◆ Inference :

$$\text{Given } X = \{x_1, x_2, \dots, x_n\}$$

$$p(X) = p(x_1, x_2, \dots, x_n)$$

$$= \int p(x_1, x_2, \dots, x_n | u) p(u) du$$

$$= \int p(x_1 | u) p(x_2 | u) \dots p(x_n | u) p(u) du$$

$$\propto |\sigma I|^{-\frac{n}{2}} |\varepsilon I|^{-\frac{1}{2}} \left( \frac{n}{\sigma} + \frac{1}{\varepsilon} \right) I \Big|^{-\frac{1}{2}}$$

$$\exp \left\{ -\frac{1}{2\sigma} \left\{ \sum_i \|x_i\|^2 - \frac{n^2 \varepsilon}{n\varepsilon + \sigma} \|\bar{x}\|^2 \right\} \right\}$$

# Training process - Local

- Local training

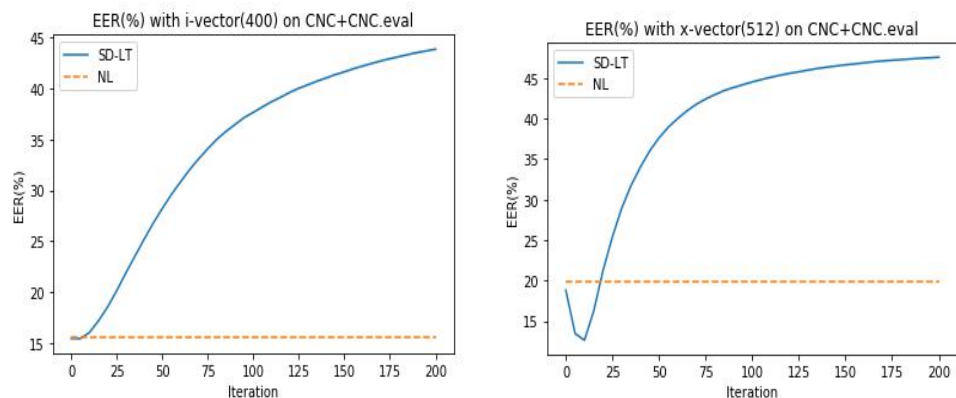
- MLLR  $x' = Mx$

$$\begin{aligned} L(M) &= \prod_c \prod_{i=1}^{n_c} \int p_l(x_i^c | u, \Sigma') p_g(u | x_1^c, \dots, x_{n_c}^c) du \\ &= \prod_c \prod_{i=1}^{n_c} \int p_g(Mx_i^c | u, \sigma I) p_g(u | x_1^c, \dots, x_{n_c}^c) du \\ &= \prod_c \prod_{i=1}^{n_c} N(Mx_i^c; \frac{n_k \varepsilon}{n_k \varepsilon + \sigma} \bar{x}_k, I(\sigma + \frac{\varepsilon \sigma}{n_k \varepsilon + \sigma})) \end{aligned}$$



# Basic results

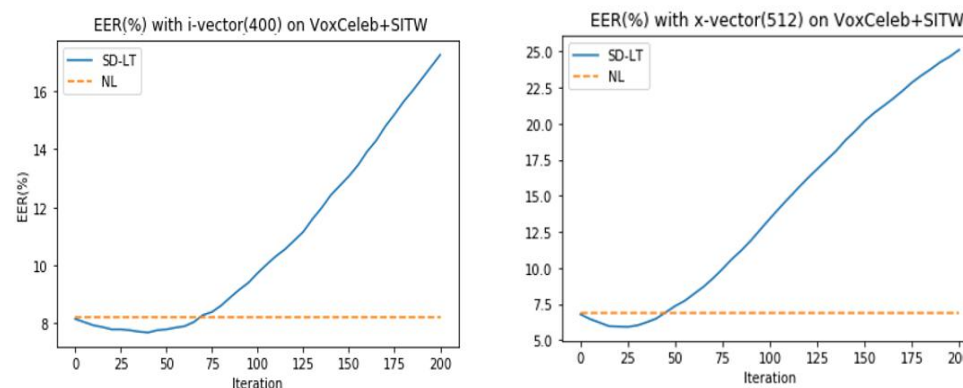
- Results on CNC.eval



EER(%) results on CNC.eval

	SD-LT (best)	NL
<i>i-vector</i>	15.44%	15.56%
<i>x-vector</i>	12.63%	19.79%

- Results on SITW



EER(%) results on SITW

	SD-LT (best)	NL
<i>i-vector</i>	15.44%	15.56%
<i>x-vector</i>	12.63%	19.79%

# Statistics analysis

- Changes in kurtosis and skewness during training of iVector and XVector on CNC (dev set and test set).

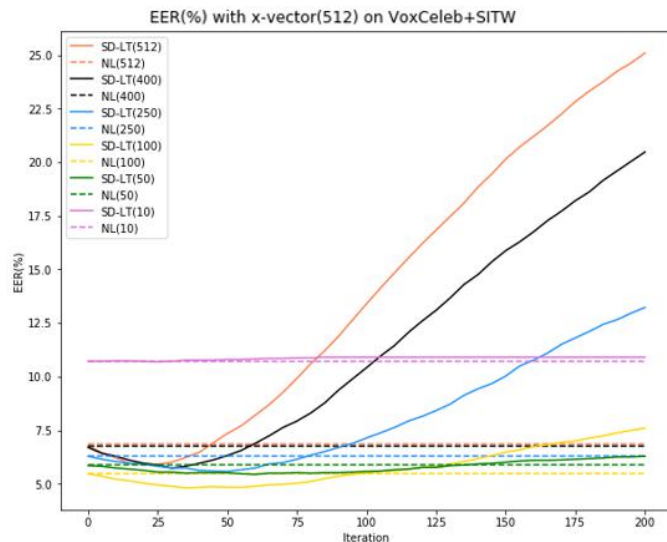
<b>dev set</b>	<b>skew</b>	<b>kurt</b>
iVector	0.001239	0.303497
xVector	-0.001600	0.300418

<b>test set</b>	<b>skew</b>	<b>kurt</b>
iVector	0.003140	0.266604
xVector	0.004902	1.513297

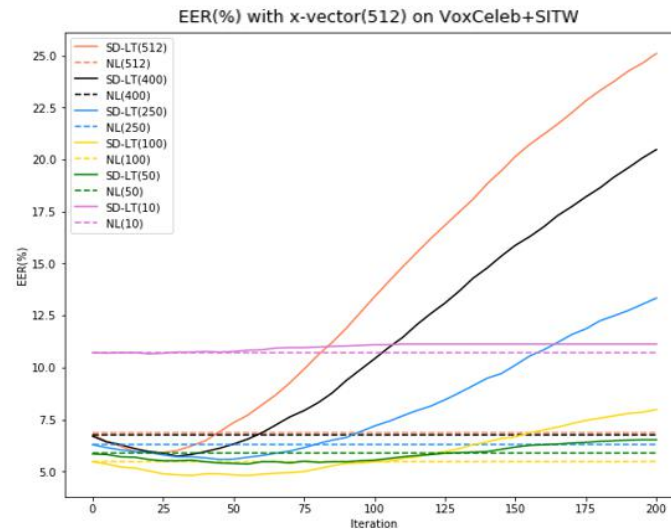
# Dimensional reduction

- The data is dimensionally reduced in two different ways and the performance in different dimensions is observed.

Dimension reduction in grading



Dimension reduction in transforming



	Dims (best/basic)	EER
i-vector on SITW	100/ <u>400</u>	7.11%/ <u>7.68%</u>
x-vector on SITW	100/ <u>512</u>	4.81%/ <u>5.91%</u>
i-vector on CNC.eval	250/ <u>400</u>	15.39%/ <u>15.44%</u>
x-vector on CNC.eval	250/ <u>512</u>	12.15%/ <u>12.63%</u>

# Relation to Length-norm(LN)

- The relationship between NL, SD/LT and standard LN was compared.

EER(%) with x-vector(512) on CNC.eval

	Optimal results(EER)
Basic NL	19.79%
+LN	12.71%
+SD/LT	<u>12.63%</u>

EER(%) with x-vector(512) on SITW

	Optimal results(EER)
Basic NL	6.86%
+LN	<u>4.51%</u>
+SD/LT	5.91%

EER(%) with i-vector(400) on CNC.eval

	Optimal results(EER)
Basic NL	15.56%
+LN	15.69%
+SD/LT	<u>15.44%</u>

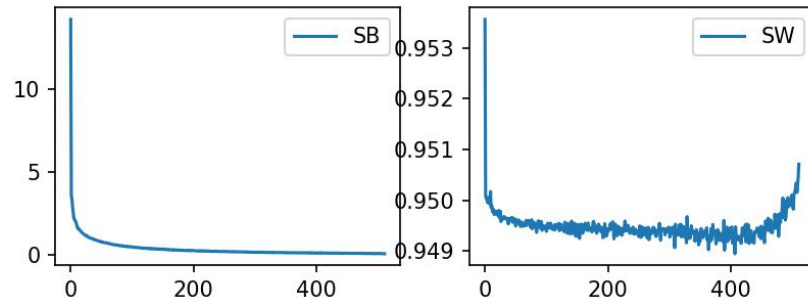
EER(%) with i-vector(400) on SITW

	Optimal results(EER)
Basic NL	8.20%
+LN	<u>6.32%</u>
+SD/LT	7.68%

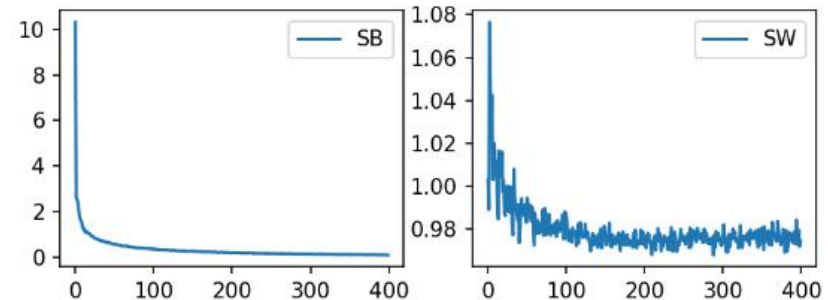
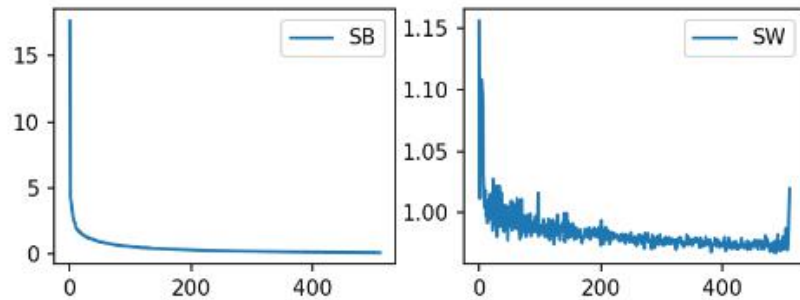
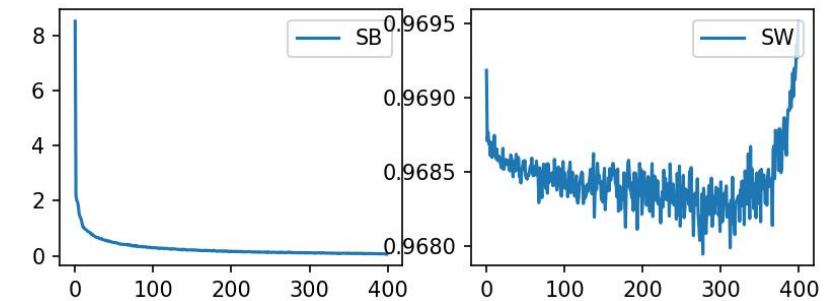
# Statistics analysis

- Why length-norm doesn't perform better than SD/LT.

*x*-vector(512) on **CNC**



*i*-vector(400) on **CNC**

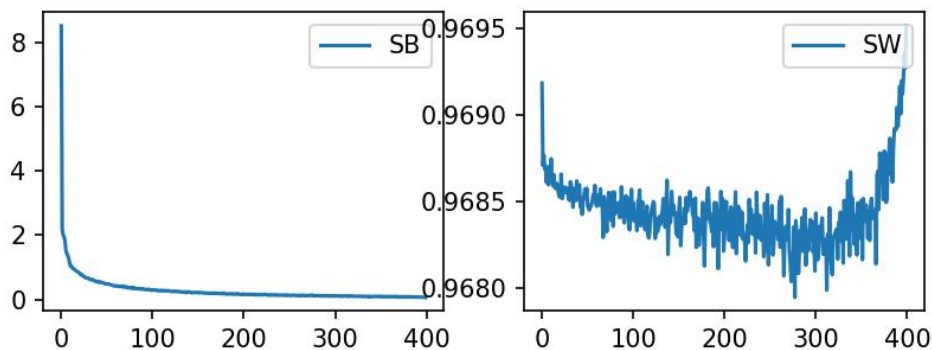


first line : SD/LT

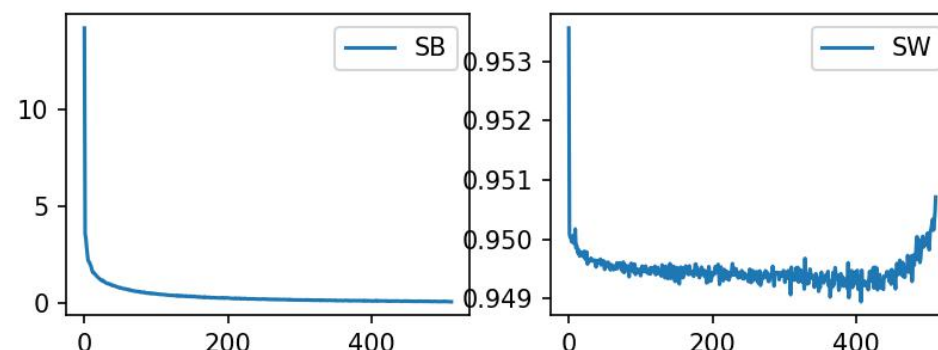
second line : length-norm

# Is within-var really equal to 1?

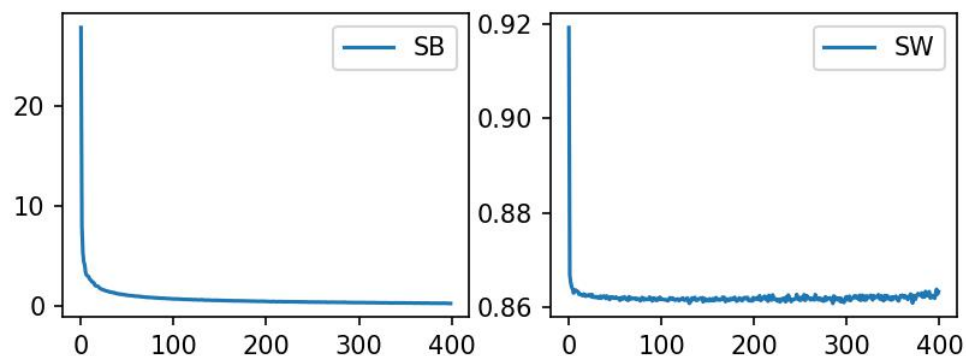
i-vector(400) on CNC



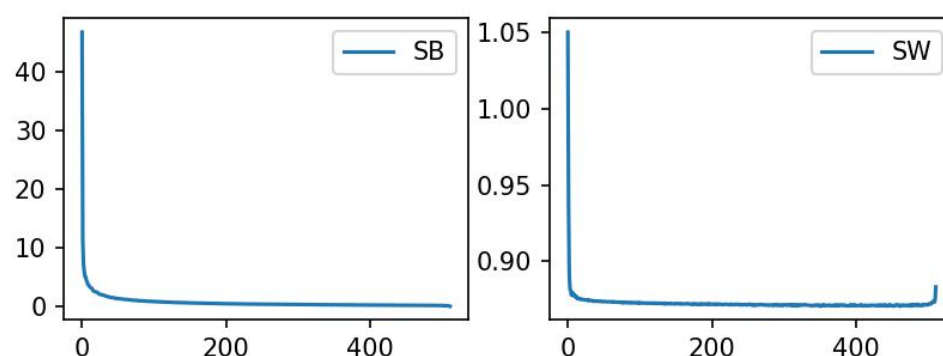
x-vector(512) on CNC



i-vector(400) on Voxceleb



x-vector(512) on Voxceleb



# Theoretical vs. statistical

- Considering that within-var is assumed to be equal to 1 in the scoring, which is not rigorous, so we replace within-var in the scoring of different experiments.

EER(%) with **x-vector**(512) on **CNC.eval**

	Optimal(EER)		Optimal (EER)
Basic(NL)	19.79%		
+LN	12.71%	SD/LT	12.63%
+*LN	12.60%	<b><u>*SD/LT</u></b>	<b><u>11.95%</u></b>

EER(%) with **x-vector**(512) on **SITW**

	Optimal(EER)		Optimal (EER)
Basic(NL)	6.86%		
+LN	4.51%	SD/LT	5.91%
+*LN	4.43%	<b><u>*SD/LT</u></b>	<b><u>4.27%</u></b>

EER(%) with **i-vector**(400) on **CNC.eval**

	Optimal(EER)		Optimal (EER)
Basic(NL)	15.56%		
+LN	15.69%	SD/LT	15.44%
+*LN	15.70%	<b><u>*SD/LT</u></b>	<b><u>15.44%</u></b>

EER(%) with **i-vector**(400) on **SITW**

	Optimal(EER)		Optimal (EER)
Basic(NL)	8.20%		
+LN	6.32%	SD/LT	7.68%
+*LN	6.29%	<b><u>*SD/LT</u></b>	<b><u>6.12%</u></b>

# Combine LN+SD/LT

- Combine LN+SD/LT and what will the performance be papered

EER(%) with *x-vector*(400) on *CNC.eval*

	Optimal(EER)
Basic(NL)	19.79%
LN	12.71%
LN +SD/LT	12.71%

EER(%) with *i-vector*(512) on *CNC.eval*

	Optimal(EER)
Basic(NL)	15.56%
LN	15.69%
LN +SD/LT	15.69%

EER(%) with *x-vector*(512) on *SITW*

	Optimal(EER)
Basic(NL)	6.86%
LN	4.51%
LN +SD/LT	4.54%

EER(%) with *i-vector*(400) on *SITW*

	Optimal(EER)
Basic(NL)	8.20%
LN	6.32%
LN +SD/LT	6.18%



# Summary

- SD/LT works better than basic NL scoring.
- X-vector performs better than i-vector.
- Length-norm doesn't do very well on the complex data set.
- It is effective to replace theoretical statistics with actual statistics and has the best performance on SD/LT.
- Length-norm + SD/LT is useless.

**Thank you !**